

Scalar scenarios contributing to $(g - 2)_\mu$ with enhanced Yukawa couplings

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Abstract

In this work we address contributions from scalars to $(g - 2)_\mu$. In order to explain the recently measured deviation by the BNL experiment on $(g - 2)_\mu$, it is necessary that these scalars are either light or couple strongly with muons. Here we explore this last possibility. We show that a scalar with mass of the order of 10^2 GeV provides significant contribution to $(g - 2)_\mu$ if the Yukawa coupling is about 10^{-1} . We suggest scenarios where this comes about naturally.

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I. INTRODUCTION

A new measurement of the muon anomalous magnetic moment, $(g - 2)_\mu$, was recently announced which indicates a deviation from the theoretical value of 2.6 sigma [1]

$$a_\mu^{exp} - a_\mu^{SM} = 426 \pm 165 \times 10^{-11}. \quad (1)$$

If this result persists [2] it implies an exciting window requiring new physics beyond Standard Model (SM). In order to explain such a deviation, various scenarios have been proposed: supersymmetry, new gauge bosons, leptoquarks, etc [3,4]. So far, SM extensions in the scalar sector have been almost neglected, which is at least plausible since in the SM the interactions between the muon and the neutral Higgs, H ,

$$f_{\mu\mu}\bar{\mu}\mu H, \quad (2)$$

gives the following contribution to $(g - 2)_\mu$ [5],

$$a_H^\mu = \frac{f_{\mu\mu}^2 m_\mu^2}{12\pi^2 m_H^2}. \quad (3)$$

Here, $f_{\mu\mu}$ is the usual Yukawa coupling for the muon, and has the following form

$$f_{\mu\mu} = \frac{m_\mu}{v_w}, \quad (4)$$

where, $m_\mu = 0.105$ GeV, is the muon mass and $v_w = 247$ GeV is the vacuum expectation value (vev) of the scalar doublet in the SM. These lead to,

$$f_{\mu\mu} \sim 10^{-3}. \quad (5)$$

With this value for $f_{\mu\mu}$ and considering the Higgs mass of the order of hundreds of GeV, $m_H \sim 10^2$ GeV, the standard Higgs contribution to $(g - 2)_\mu$ is negligible,

$$a_H^\mu \sim 10^{-13}. \quad (6)$$

The previous analysis is general enough to be extended to other scalar scenarios. However, if we want to generate $a^\mu \sim 10^{-9}$ through scalars, we have to consider either a light scalar of mass around $m_H \sim 10$ GeV and $f_{\mu\mu} \sim 10^{-2}$, or keep $m_H \sim 10^2$ GeV and take $f_{\mu\mu} \sim 10^{-1}$, which demands a vev, $v \sim 1$ GeV. This last possibility is very suggestive, since a VEV of the order of few GeV's is a natural choice when this scalar is in charge of generating the charged lepton masses, and the heaviest lepton, the tau, has mass $m_\tau \sim 1.7$ GeV. In view of this we expect that economic modifications on the scalar sector in SM can lead to such a favorable configuration of parameters.

Motivated by this we want to suggest scenarios where this can come about, like minimal extensions in the scalar sector of SM itself (adding more scalars), as well as a model that requires small vev's, like $3 - 3 - 1$ [6].

In what follows, we will present the scenarios we have in mind. In section II we extend the SM scalar sector by adding a doubly charged Higgs boson, which interacts solely with right handed charged leptons. We compute its contribution to $(g - 2)_\mu$ for a range of masses

fixing the Yukawa coupling, so that we can choose the appropriate mass for the pointed deviation in $(g - 2)_\mu$. In section III we introduce a second Higgs doublet interacting with leptons only, which generates the charged lepton masses. We present the potential for the scalars and, considering the constraints coming from it, as well some suitable choices of the remaining parameters, we are able to find the mass which best fits the deviation. We then embed, in section IV, the previous scenarios in a $3 - 3 - 1$ model. Finally, we present some concluding remarks in section V.

II. DOUBLY CHARGED SCALAR

We first consider a minimal extension of the scalar sector in the SM in order to accommodate a doubly charged scalar singlet η^{++} . We also attribute to it two units of lepton number $L = L_e + L_\mu + L_\tau = -2$ and hypercharge $Y = 4$, so that it interacts only with the right-handed leptons as follows,

$$\mathcal{L} = h_{ab} l_{aR} C l_{bR} \eta^{++}, \quad (7)$$

where C is the charge conjugation matrix in some representation. This interaction provides six additional contributions to $(g - 2)_\mu$ besides those from SM.

However in what follows we just take into account the contributions that conserve flavor, i.e., those which involve the coupling $h_{\mu\mu}$. This seems a natural assumption since these contributions are expected to dominate over the off-diagonal ones. Let us postpone the discussion about the lepton flavor mixing terms and compute the $(g - 2)_\mu$ for the diagonal term only, assuming a strongly coupled η^{++} to leptons. The two contributions considered here are depicted in Figure (1), and can be expressed respectively by,

$$\begin{aligned} a) \quad a_\mu^\eta &= \frac{-h^2}{2\pi^2} \int_0^1 \frac{x^3 - x^2}{x^2 + (z - 2)x + 1}, \\ b) \quad a_\mu^\eta &= \frac{h^2}{4\pi^2} \int_0^1 \frac{x^2 - x^3}{x^2 + z(1 - x)}, \end{aligned} \quad (8)$$

where $z = \frac{m_\eta^2}{m_\mu^2}$ and $h = h_{\mu\mu}$.

Considering $h \simeq 1$, we observe that the measured deviation $a_\mu \sim 10^{-9}$ favors such a doubly charged scalar with a mass $m_\eta \sim 200$ GeV. This is very interesting, since the addition of a singlet scalar is the simplest modification we can imagine in the scalar sector of the SM. Of course, one could imagine such a scalar as an ingredient in some classes of models dealing with more fundamental questions besides the muon anomalous magnetic moment. It is reasonable, then, to expect that this economic addition to SM could be embedded in a larger structure, and it is in this context that we hope this extension plays an important role, although here we only worried in suggesting a picture where heavy scalars would be important for the $(g - 2)_\mu$ deviation.

FIGURES

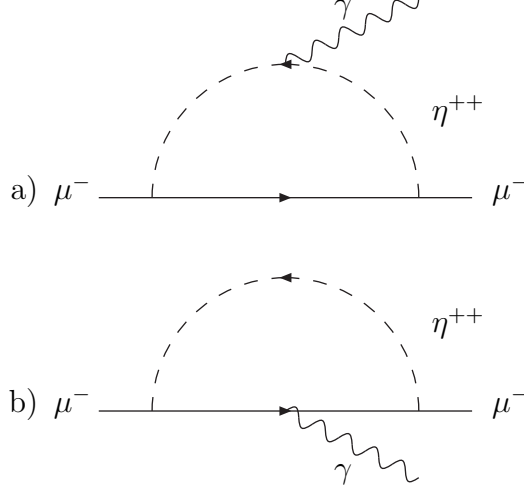


FIG. 1. Doubly charged scalar contributions to the muon anomalous magnetic moment $(g - 2)_\mu$.

We can turn now to the point concerning the assumption we made over the off-diagonal couplings in Eq.(7). Those interactions clearly lead to flavor changing processes by exchanging η^{++} . Such processes severely constrain the off-diagonal components of the Yukawa coupling matrix, h_{ab} for $a \neq b$. Let us consider only three of the flavor changing process¹ : $\mu \rightarrow 3e$, $\tau \rightarrow 3\mu$, $3e$. The decay rate of a lepton, l' , in three lighter leptons, l , allowed by the interaction in Eq. (7) has, in general, the following expression [7]

$$\Gamma(l' \rightarrow 3l) \simeq \frac{h_{l'l}^2 h_{ll}^2 m_{l'}^5}{192\pi^3 m_\eta^4}. \quad (9)$$

The present experimental data on these flavor changing processes are: $BR(\mu \rightarrow 3e) \lesssim 10^{-12}$, $BR(\tau \rightarrow 3e, 3\mu) \lesssim 10^{-6}$ [8]. This can be translated to the following constraints: $\frac{h_{e\mu}h_{ee}}{m_\eta^2} \lesssim 10^{-11} \text{ GeV}^{-2}$ and $\frac{h_{e\tau}h_{ee}}{m_\eta^2}, \frac{h_{\mu\tau}h_{\mu\mu}}{m_\eta^2} \lesssim 10^{-7} \text{ GeV}^{-2}$. If we have a scalar with mass, $m_\eta \simeq 10^2 \text{ GeV}$, these constraints require: $h_{e\mu}h_{ee} \lesssim 10^{-7}$ and $h_{e\tau}h_{ee}, h_{\mu\tau}h_{\mu\mu} \lesssim 10^{-3}$. Hence in order to explain the $(g - 2)_\mu$ with heavy scalars, and consequently, enhanced diagonal Yukawa couplings, h_{aa} , we can safely assume $h_{a \neq b} \simeq 0$. By the other hand, if we demand that the interaction in Eq.(7) respects all the global symmetries of the standard model, we automatically have $h_{a \neq b} = 0$ since the off-diagonal components of h violate the global symmetries $U(1)_{L_e, L_\mu, L_\tau}$.

Concerning the diagonal components, there is a lower bound to the product of h_{ee} and $h_{\mu\mu}$ imposed by muonion-antimuonion conversion: $\frac{h_{ee}h_{\mu\mu}}{m_\eta^2} > 10^{-8} \text{ GeV}^{-2}$ [9]. For $m_\eta \simeq 10^2 \text{ GeV}$

¹Other rare leptonic decays, for each lepton flavor decaying in three leptons, are of the same order of magnitude as those presented here and would lead to the same constraints.

we have $h_{ee}h_{\mu\mu} > 10^{-4}$. Along with this the only upper bounds come from $(g-2)_e$ and the Bhabha scattering process. From these, the last is the most stringent [10]: $\frac{h_{ee}^2}{m_\eta^2} < 10^{-6} \text{ GeV}^{-2}$, which requires, in our case, $h_{ee} < 10^{-1}$. There is no experimental constraint on $h_{\mu\mu}$, except for the recent $(g-2)_\mu$ deviation, which can be solved, as in the above proposal, with $h_{\mu\mu} \simeq 1$ for $m_\eta \simeq 10^2 \text{ GeV}$.

III. A SECOND SCALAR DOUBLET

The second scenario we consider here is the one where a second Higgs doublet,

$$H_1 = \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix}, \quad (10)$$

is added to the SM. Let us assume that this Higgs is in charge of generating charged lepton masses only,

$$\mathcal{L}^Y = f_{ii} \bar{L}_{iL} H_1 l_{iR} + h.c. \quad (11)$$

Now, consider the following potential for the two Higgs doublet:

$$V = \frac{\mu_1^2}{2} H_1^2 + \frac{\mu_2^2}{2} H_2^2 + \frac{\lambda_1}{4} H_1^4 + \frac{\lambda_2}{4} H_2^4 + \frac{\lambda_3}{2} H_1^2 H_2^2 - \mu H_1 H_2. \quad (12)$$

Looking for the minimum of this potential we have the following constraints,

$$\begin{aligned} v_1(\mu_1^2 + \lambda_1 v_1^2 + \lambda_3 v_2^2) - \mu v_2 &= 0, \\ v_2(\mu_2^2 + \lambda_2 v_2^2 + \lambda_3 v_1^2) - \mu v_1 &= 0. \end{aligned} \quad (13)$$

Taking

$$\mu_2^2 < 0, \quad \mu_1^2 > 0, \quad \mu \ll \mu_2^2, \quad (14)$$

and considering that the second Higgs doublet H_2 is in charge of the quark masses, the value of its vev must be around $v_2 \sim 10^2 \text{ GeV}$. In this case, we have

$$v_1 \sim \frac{\mu v_2}{\mu_1^2 + \lambda_3 v_2^2}, \quad v_2^2 \sim -\frac{\mu_2^2}{\lambda_2}. \quad (15)$$

Assuming $\mu_1 \sim 10^2 \text{ GeV}$, and $\mu \sim 10^2 \text{ GeV}^2$, we find

$$v_1 \sim 1 \text{ GeV}, \quad (16)$$

which gives $f_{\mu\mu} = \frac{m_\mu}{v_1} \sim 0.1$, leading to the expected contribution to $(g-2)_\mu$,

$$a^\mu \sim 10^{-9}. \quad (17)$$

A scheme like this was recently suggested by Ma and Raidal in two different scenarios. In the first one [4,11], the Higgs doublet H_1 carries lepton number and is used to generate neutrino masses, for which they need a vev, $v_1 \sim 1 \text{ MeV}$, which demands a Higgs, H_1 , with

mass around few TeV's. In the second scenario [12], this Higgs doublet is associated with a global symmetry $U(1)$ which only permits the doublet to interact with the light quarks, delegating to H_1 the role of generating their masses. Such a scenario could as well be realized in the leptonic sector, as we suggested above.

In principle this case also leads to flavor changing processes whose sources are the interactions: $f_{ij}\bar{\nu}_{iL}e_{jR}H^+$ for $i \neq j$. However, it is possible to avoid these processes by assuming lepton number conservation under the symmetry $U(1)_{L_e, L_\mu, L_\tau}$. In this case the existing experimental data are not sufficient to give any constraint over this scenario. The reason is that a considerable enhancement of Yukawa couplings only happens for the heaviest leptons, namely, $f_{\mu\mu}$ and $f_{\tau\tau}$. The enhancement of f_{ee} is not sensible enough to be prompted in an electron collider. Nevertheless, a scenario like this can be appropriately tested in a muon collider, where simple processes like $\mu^+\mu^- \rightarrow \mu^+\mu^-$, $\bar{b}b$ are able to assess such enhanced Yukawa couplings. An analysis in this direction is done in Ref. [13].

IV. EMBEDDING OF BOTH SCENARIOS IN A 3 – 3 – 1 GAUGE THEORY

Now let us think about a model which naturally accommodates both scenarios presented above. Such a candidate could be the model based in the 3 – 3 – 1 symmetry [14]. In its minimal version the masses of charged leptons and neutrinos are generated by a sextet of scalars,

$$S = \begin{pmatrix} \sigma_1 & \frac{h_2^-}{\sqrt{2}} & \frac{h_1^+}{\sqrt{2}} \\ \frac{h_2^-}{\sqrt{2}} & H_1^{--} & \frac{\sigma_2}{\sqrt{2}} \\ \frac{h_1^+}{\sqrt{2}} & \frac{\sigma_2}{\sqrt{2}} & H_2^{++} \end{pmatrix}. \quad (18)$$

After the breaking of the 3 – 3 – 1 symmetry to the standard 3 – 2 – 1 symmetry, this sextet will decompose under 3 – 2 – 1 in the following triplet, doublet and singlet of scalars, respectively [6,15]:

$$\Delta = \begin{pmatrix} \sigma_1 & \frac{h_2^-}{\sqrt{2}} \\ \frac{h_2^-}{\sqrt{2}} & H_1^{--} \end{pmatrix}, \quad \Phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} h_1^+ \\ \sigma_2 \end{pmatrix}, \quad H_2^{++}. \quad (19)$$

Their Yukawa interactions with leptons are

$$\mathcal{L}^Y = f_{ii}\bar{L}_{iL}^c\Delta L_{iL} + f_{ii}\bar{L}_{iL}\Phi_3 l_{iR} + f_{ii}\bar{l}_{iR}^c l_{iR} H_2^{++}. \quad (20)$$

Note that the last two interactions in Eq.(20) account for the two scenarios previously discussed.

It was shown in Ref. [6] that the vev of the scalars σ_1 and σ_2 are related by

$$v_{\sigma_1} = \frac{M v_{\sigma_2}^2}{v_\chi^2}, \quad (21)$$

where M is a free parameter with mass dimension, and v_χ is the vev of the triplet χ which breaks the symmetry 3 – 3 – 1 to 3 – 2 – 1.

Observe that v_{σ_1} and v_{σ_2} have the same origin, the sextet. Then we would expect that both take almost the same value. Nevertheless, one is responsible for the neutrino masses, and then should be of the order of eV, and the other is responsible by the charged lepton masses, which are of the order of GeV. Then it is reasonable to set the value of v_{σ_2} around few GeV's. In Eq.(21), v_χ is the vev of the scalar triplet χ , which sets the scale of $3 - 3 - 1$ breaking, around TeV, and M comes from a term in the potential that violates explicitly the lepton number conservation and should be small. So, taking the set of values used in Ref. [6], $v_{\sigma_2} = 1$ GeV, $v_\chi = 10$ TeV and $M = 0.1$ GeV we obtain $v_{\sigma_1} = 1$ eV, which automatically provides an explanation for the pointed deviation in $(g - 2)_\mu$ and also leads to the expected scale for neutrino masses (see Ref. [16] for other sources contributing to $(g - 2)_\mu$ inside $3 - 3 - 1$ models).

V. CONCLUSIONS

In this work we showed that contributions from scalars with masses around hundreds of GeV's can contribute significantly to $(g - 2)_\mu$, and have potential capacity of explaining its theoretical deviation from the observed BNL measurement. We suggested two scenarios where such scalars would appear naturally, and a model where they are a constituent part, the $3 - 3 - 1$ gauge model. Besides, both scenarios often appear in models beyond SM, like neutrino mass models, Left-Right models, grand unified theories, etc., and so are well motivated.

We should stress though that our analysis is based on one-loop calculations only. It was shown that in some models containing scalars (pseudo-scalars), two loop contributions to $(g - 2)_\mu$ can be even more significant than the one loop contribution (see Ref. [17] for a detailed discussion). This is due to the Barr-Zee two loop diagram [18]. Essentially what happens is the following: from the Barr-Zee diagram comes a factor m_f/m_μ which can enhance the two loop contributions in relation to the one loop when a heavy fermion, f , is flowing in the inner loop. The enhancement also depends on other parameters as $\tan \beta$ and the pseudo-scalar mass, M_a , involved in the loop. According to Ref. [17] the Barr-Zee type contribution gives a good fit to the recent 2.6 sigma deviation when $\tan \beta \simeq 50$ and $M_a < 40$ GeV. It is not obvious that such a conclusion would survive for the two Higgs doublet scenario studied here, since these parameters have values a little diverse when roughly set according to our scenario, $\tan \beta \simeq 10^2$ (according to the vev's assumed in section III) and $M_a \simeq 10^2$ GeV. It would be necessary a thoroughly new calculation to achieve any additional result besides those already presented in section III, which we cannot assure will be relevant. In summary, the simple scenarios we proposed for heavy scalars and enhanced Yukawa couplings, can easily accommodate the 2.6 sigma $(g - 2)_\mu$ deviation at one loop level.

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